Maximizing Power Transfer in an Electric Circuit

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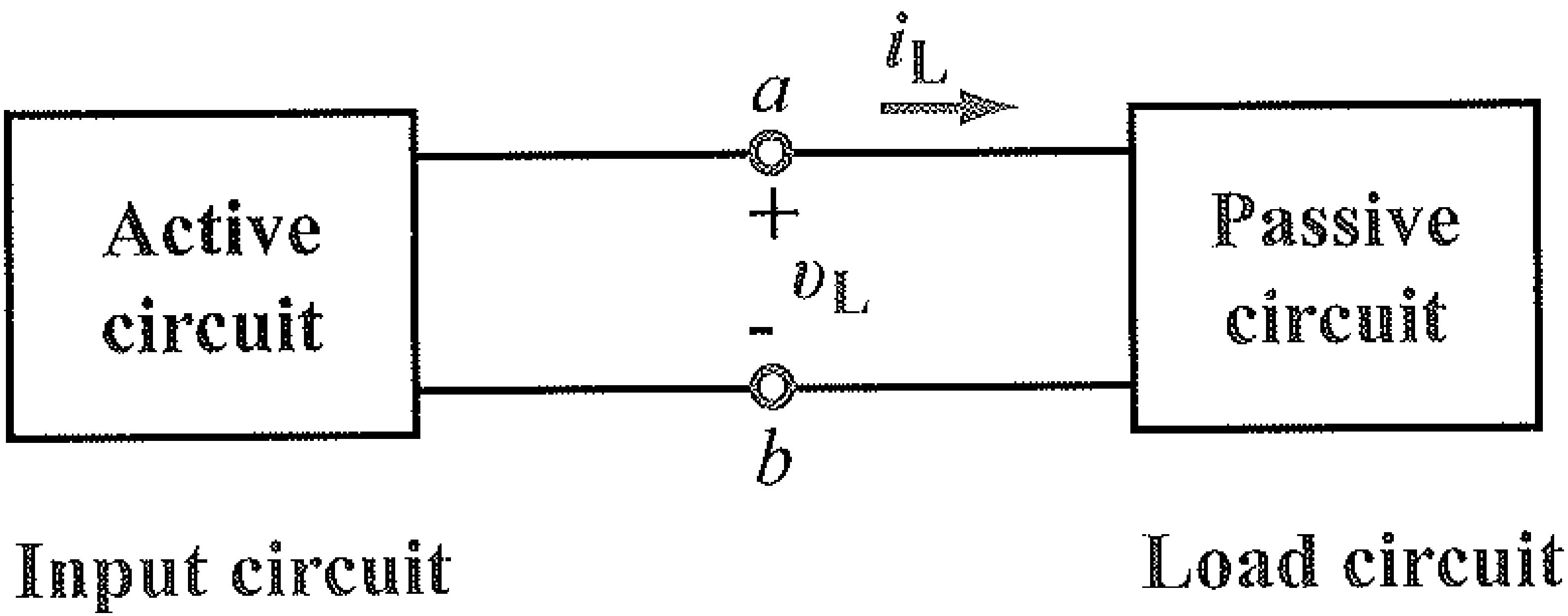
Model

- For certain applications, it is desirable to maximize the power (P_L) that is transferred from the Input circuit to the Load circuit.

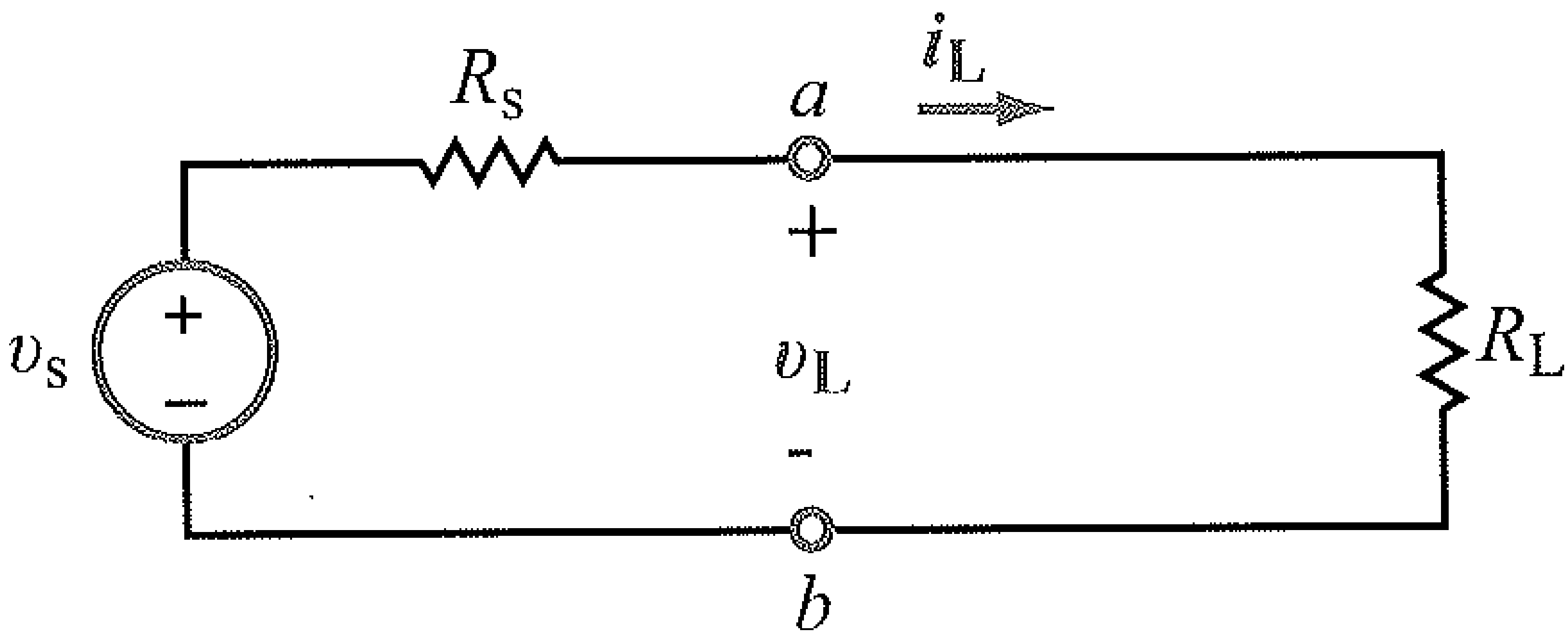


Problem

- Given a specified Load circuit how does one approach the design of the load circuit to maximize the transfer of Power to the load?

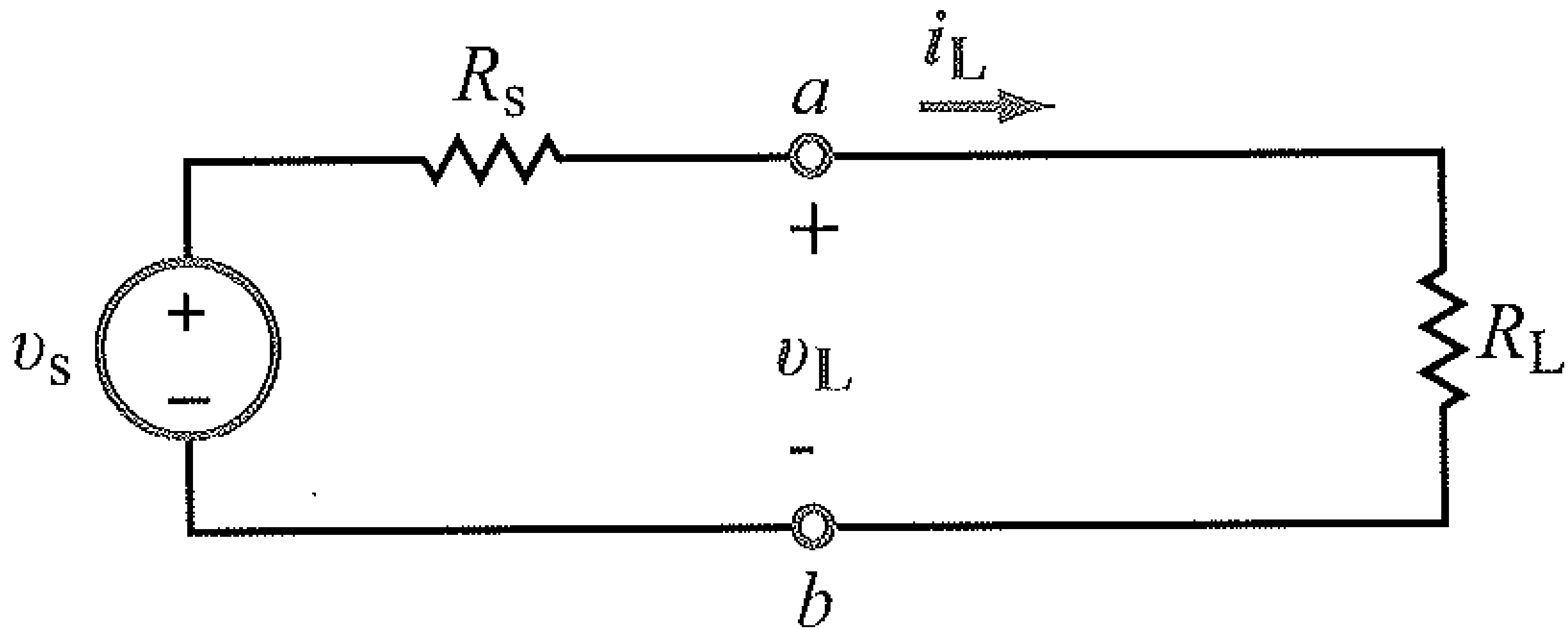
Simplified Model

- We begin by replacing the original Model with a Simplified Model.



Restatement of Problem (Based on Simplified Model)

- Find the value of R_L that achieves maximum Power transfer.

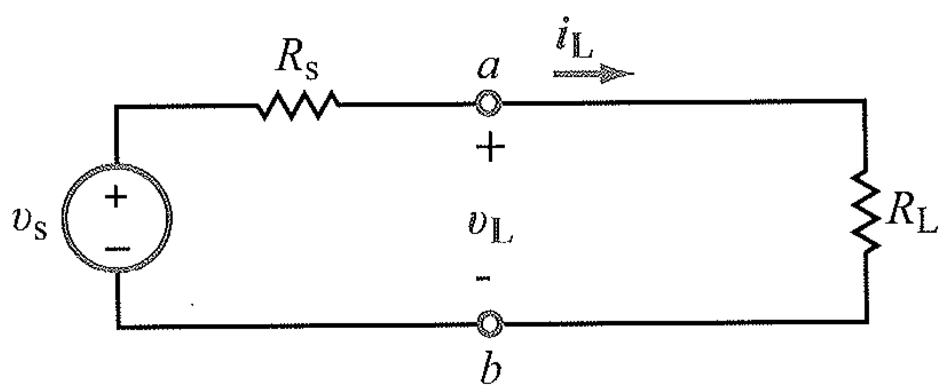


Analysis

- We first write expressions for the load current (i_L) and the load voltage (v_L).

$$i_L = \frac{v_s}{R_s + R_L}$$

$$v_L = \frac{v_s R_L}{R_s + R_L}$$



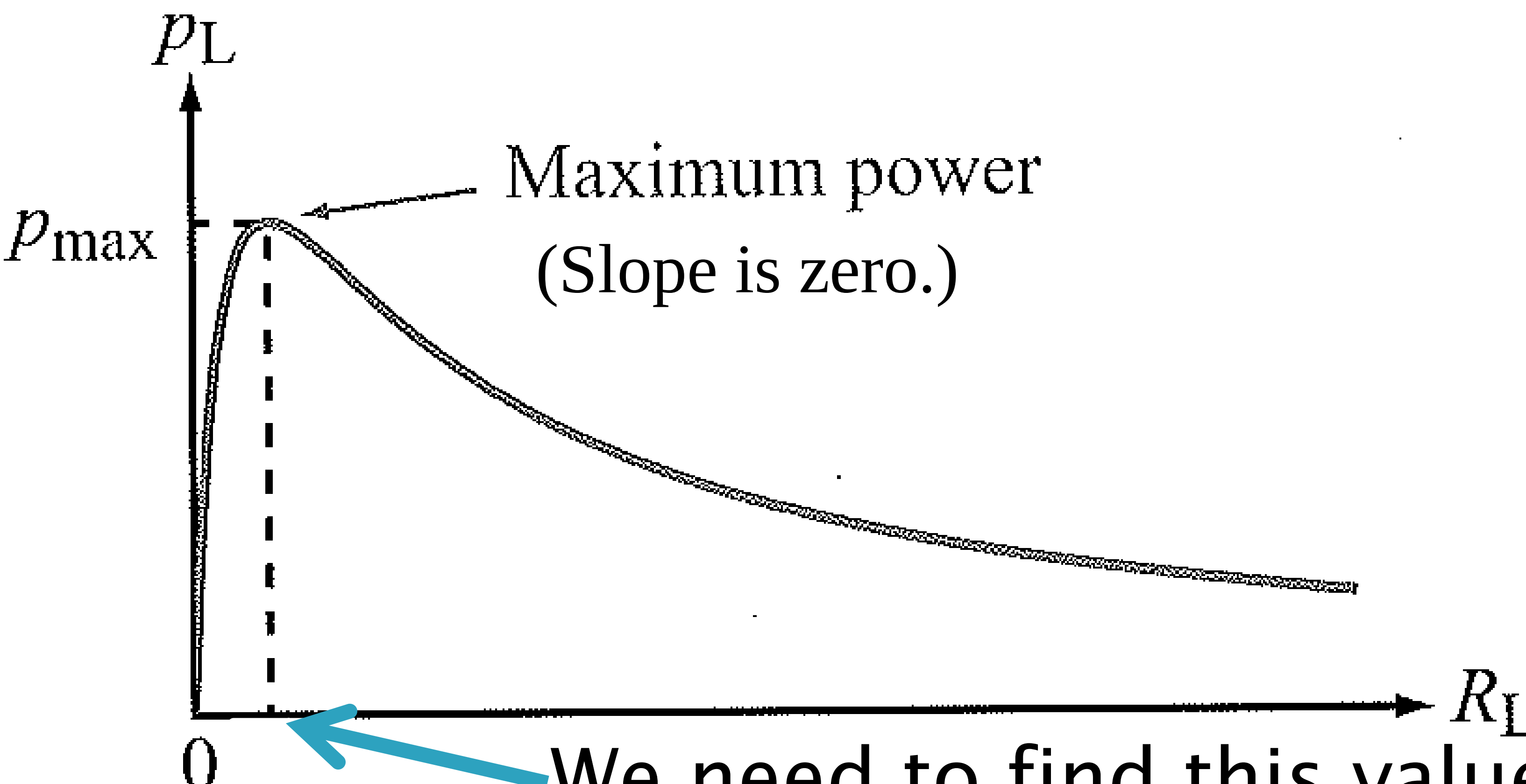
Power Transferred to the Load

- The power (PL) that is transferred to the Load can be expressed as

$$P_L = i_L \; v_L = \frac{v_s \; R_L}{\left(R_s + R_L\right)^2}$$

Power as a Function of R_L

- The expression for the Power transferred to the Load is a nonlinear function of R_L.



We need to find this value!

Derivative

- The derivative of P_{L} with respect to R_{L} is:

$$\frac{d\,P_L}{d\,R_L} = v_s\,\frac{\left(R_s + R_L\right)^2 - 2\,R_L\left(R_s + R_L\right)}{\left(R_s + R_L\right)^4}$$

- The derivative is zero when the numerator is zero. Thus we must find the value for R_{L} that satisfies the equation:

$$\left(R_x + R_L\right)^2 - 2\,R_L\left(R_s + R_L\right) = 0$$

$$\left(R_s + R_L\right)^2 - 2R_L\left(R_s + R_L\right) = 0$$

$$\left(R_s + R_L\right) - 2R_L = 0$$

$$\left(R_s + R_L\right) = 2R_L$$

$$\boxed{R_s = R_L}$$

So, maximum power is transferred to the load when the load has a resistance equal to the resistance of the source!

Summary

- The condition that should be met to achieve maximum power transfer is

$$R_L = R_s$$

- When this condition is met, the maximum power can be expressed as

$$P_{\max} = \frac{{v_s}^2 R_L}{(R_s + R_L)^2} = \frac{{v_s}^2}{4R_L}$$